Wage Adjustment: A Network Approach

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Introduction

- Wage adjustment is a central question
  - For instance, sluggish wages cause underemployment during recessions
- Most of the existing literature focuses on nominal rigidities.
- We study the role of strategic interactions, which is a **real rigidity** (Berger et al., 2022)
- We use network methods to address Manski's reflection problem (and other identification problems).
- **Key questions**:
  - How strong are strategic interactions in wage setting?
  - What are the macroeconomic implications for wage adjustment?

This paper

- Study an environment where firms adjust wages due to “exogenous” reasons
  - ... the Colombian economy following the 2014-2015 exchange rate depreciation Figure
  - ... “revenue” shocks for export-oriented firms Figure
  - ... “cost” shocks for import-oriented ones Figure
- We map competitors in the labor market using worker flows between firms
  - Developments in networks econometric (peer effects estimation)
  - Exploit granular administrative employee-employer data

Preview of Results

- Strong degree of strategic interactions in wage setting
  - **Endo strategic interactions**: Firms react to wage adjustment by their competitors (**0.2-0.3**)
  - **Exo strategic interactions**: Firms react to external shocks to their competitors
- Strategic interactions slow down aggregate wage adjustment by **10 p.p.** over two years
- Real rigidities lead to slower adjustment than nominal rigidities.

Roadmap

- Model highlights
- Empirical Analysis
    - Data
    - Strategic Interactions in Wage Setting

Model

Model: Overview

- Main ingredients are *Workers* and *Firms*
  - Wage posting model
  - $\mathcal{X}$ types of workers. Each type $x \in \{1, \ldots, \mathcal{X}\}$ is of unit mass.
  - J firms are grouped into sets, $g = 0, 1, \ldots, \mathcal{G}$.
  - Set of firms in group g is $\mathcal{J}_g$.
- Closing the Model
  - Final good producers Link
  - Households Link
  - Open-economy block Link
  - Equilibrium Link
- Calibration Link

Model: Labor Supply

- Labor supply to firm j (probability choosing j):

$$L_{xjt}\left(W_{xjt}, \mathbb{W}_{-xjt}; Z_{jt}, \mathbb{Z}_{-jt}\right) = \frac{\left(W_{xjt}^{\phi_x} V_{xj} {Z_{jt}}^{\beta_z}\right)^{\frac{1}{1-\rho_x}}}{D_{xgt}} L_{xgt}, \tag{1}$$

  - W_{xjt} is wage of firm j at t
  - ϕ_x is degree of substitution across firms
  - V_{xj} is "amenities" of firm j
  - Z_{jt} is firm j's external shock(s) at t
  - where $\mathbb{W}_{-xjt} \equiv \{W_{xkt}\}_{k \neq j}$ and $\mathbb{Z}_{-jt} \equiv \{Z_{kt}\}_{k \neq j}$
  - $\log D_{xgt}$ is inclusive value of group g: $D_{xgt} = \sum_{k \in \mathcal{J}_g} \left({W_{xkt}}^{\phi_x} V_{xk} {Z_{kt}}^{\beta_z}\right)^{\frac{1}{1-\rho_x}}$
  - L_{xgt} is labor supply to group g
  - Micro-foundation

Model: Firms

- Firm j maximizes their profit:

$$\mathcal{V}^f_{jt} = P_{jt} Y_{jt} + \mathcal{E}_t P^*_{jt} Y^*_{jt} - \sum_{x=1}^{\mathcal{X}} W_{xjt} L_{xjt} - \mathcal{E}_t M_{jt}, \tag{2}$$

s.t. labor supply (1)

product demand Link

capacity constraint $Y_{jt} + Y^*_{jt} \leq F\left(\{L_{xjt}\}_{x=1}^{\mathcal{X}}, M_{jt}, \alpha_j\right)$ Link

market structure $\{H_{xjkt}\}_{k \neq j \text{ and } k,j \in \mathcal{J}_g}$ Link

- P_{jt} and P^*_{jt} are prices in pesos and dollars (Gopinath et al. (2020))
- $\mathcal{E}_t$ defines exchange rate
- L_{xjt} is type-x labor
- M_{jt} is foreign intermediate inputs
- α_j is share of foreign intermediate inputs in production
- $\gamma_j \equiv \frac{\mathcal{E}_t P^*_{jt} Y^*_{jt}}{P_{jt} Y_{jt} + \mathcal{E}_t P^*_{jt} Y^*_{jt}}$ is a proxy to share of trade with abroad

Model: Equilibrium Wage

- Log-linearized equilibrium wage is determined by:

$$\log W_{xjt} = \underbrace{\log \tilde{Y}_{xjt} + (\tilde{\gamma}_j - \tilde{\alpha}_j) \log \mathcal{E}_t}_{\text{marginal value added}} + \underbrace{\log \mathcal{M}_{xjt} \left(W_{xjt}, \mathbb{W}_{-xjt}; \{\tilde{\gamma}_k \log \mathcal{E}_t, \tilde{\alpha}_k \log \mathcal{E}_t\}_{k=1}^{\mathcal{J}} \right)}_{\text{markdown}}, \quad (3)$$

- $\tilde{\gamma}_j \sim \frac{\mathcal{E}_t P_{jt}^* Y_{jt}^*}{P_{jt} Y_{jt} + \mathcal{E}_t P_{jt}^* Y_{jt}^*}$ is a proxy to share of trade with abroad $\rightarrow Z_{jt}^1 = \mathcal{E}_t^{\tilde{\gamma}_j}$
- $\tilde{\alpha}_j$ is a proxy to share of foreign intermediate inputs in production $\rightarrow Z_{jt}^2 = \mathcal{E}_t^{\tilde{\alpha}_j}$
- Strategic interactions ...

... endogenous	$\mathbb{W}_{-xjt}$
... exogenous	$\{\tilde{\gamma}_k \log \mathcal{E}_t, \tilde{\alpha}_k \log \mathcal{E}_t\}_{k=1}^{\mathcal{J}}$

Data and Empirical Analysis

Data

- Employee-employer data
  - Social security payment reports from the *PILA* system, monthly (including days worked)
  - Period: 2009 - 2018
  - Sample: $\approx$ 6 mln firm-worker per year, $\approx$ 13 mln unique workers, 0.6 mln firms and 0.743 mln MSA-firms
    - Cluster workers into $\mathcal{X}$-groups using worker pay components (Abowd et al. (1999)) Link
    - Remove workers with wages below minimum wages
    - Restrict to the largest connected sets of firms within each metropolitan area
    - Adjust for changes in firm identifiers (Benedetto et al. (2007))
  - Summary statistics Link, spatial coverage Link, industry coverage Link and heatmaps Link
- IO matrix at the region and 2-digit industry level (33 regions $\times$ 61 industries)
  - Share of trade with abroad and share of foreign intermediate inputs in production

Construction of Variables

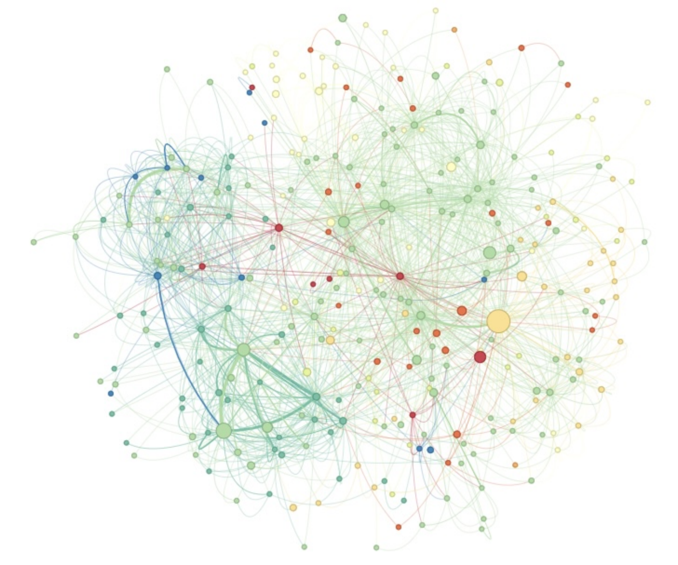
$$\Delta \log W_{xjt} = \iota_{xm} + \lambda \sum_{k \neq j} \pi_{x,jk} \Delta \log W_{xkt} + \beta_1^{\text{exp}} \tilde{\gamma}_j \Delta \log \mathcal{E}_t + \beta_1^{\text{imp}} \tilde{\alpha}_j \Delta \log \mathcal{E}_t +$$
$$\beta_2^{\text{exp}} \sum_{k \neq j} \pi_{x,jk} \left(\tilde{\gamma}_k - \tilde{\gamma}_j \right) \Delta \log \mathcal{E}_t + \beta_2^{\text{imp}} \sum_{k \neq j} \pi_{x,jk} \left(\tilde{\alpha}_k - \tilde{\alpha}_j \right) \Delta \log \mathcal{E}_t + u_{xjt}$$

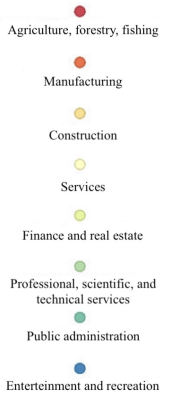
- π_x is constructed using flows of job-movers between 2013-2018

$$\pi_{x,jk} = \frac{\sum_{t=2013}^{2018} \text{Flows}_{xt,j \leftrightarrow k}}{\sum_{t=2013}^{2018} \sum_{i=1:i \neq j}^{\mathcal{J}} \text{Flows}_{xt,j \leftrightarrow i}}, \; k \neq j,$$

- ι_{xm} is an MSA fixed effect

Job flows: Illustration (Cartagena)





Construction of Variables cont'd

- Wages of job-stayers
  - First, we difference wages between $t-1$ and year t at worker level:
  - Second, aggregated to firm level for job-stayers weighting with working days
- Exogenous variables is difference between 2014 and year t:

$$\tilde{\gamma}_j \Delta \log \mathcal{E}_t = \text{Share of Trade with Abroad}_{RI(j)} \times (\log \mathcal{E}_t - \log \mathcal{E}_{2014})$$
$$\tilde{\alpha}_j \Delta \log \mathcal{E}_t = \text{Share of Foreign Inputs}_{RI(j)} \times (\log \mathcal{E}_t - \log \mathcal{E}_{2014})$$

  - where $\mathcal{E}_{2014}$ and $\mathcal{E}_t$ are average exchange rates in 2014 and year t, respectively
  - Share of Trade with Abroad$_{RI(j)}$ is share of exports to total output in region-industry RI
  - Share of Foreign Inputs$_{RI(j)}$ is share of foreign intermediate inputs in region-industry RI
  - Exposure map Figure

Identification

$$
\begin{aligned}
\Delta \log W_{xjt} = & \iota_{xm} + \lambda \sum_{k \neq j} \pi_{x,jk} \Delta \log W_{xkt} + \beta_1^{\text{exp}} \tilde{\gamma}_j \Delta \log \mathcal{E}_t + \beta_1^{\text{imp}} \tilde{\alpha}_j \Delta \log \mathcal{E}_t + \\
& \beta_2^{\text{exp}} \sum_{k \neq j} \pi_{x,jk} \left(\tilde{\gamma}_k - \tilde{\gamma}_j \right) \Delta \log \mathcal{E}_t + \beta_2^{\text{imp}} \sum_{k \neq j} \pi_{x,jk} \left(\tilde{\alpha}_k - \tilde{\alpha}_j \right) \Delta \log \mathcal{E}_t + u_{xjt}
\end{aligned}
$$

- Kuersteiner and Prucha (2020)
    - Linear and quadratic moments to identify strategic interactions
- Endogenous networks: estimate network formation
    - pseudo-maximum likelihood approach (Silva & Tenreyro, 2006)

Strategic Interactions and Wage Adjustment: Estimation Results

	2014-15	**2014-16**	2014-17	2014-18
$\pi\Delta\log W$	0.235***	**0.217*** **	0.261***	0.346***
	(0.034)	**(0.022)**	(0.016)	(0.014)
$\tilde{\gamma}\Delta\log\mathcal{E}$	0.007	**0.002**	0.257***	0.254***
	(0.037)	**(0.064)**	(0.085)	(0.070)
$\tilde{\alpha}\Delta\log\mathcal{E}$	-0.228***	**-0.875*** **	-1.263***	-1.595***
	(0.078)	**(0.135)**	(0.177)	(0.150)
$(\pi\tilde{\gamma}-\tilde{\gamma})\,\Delta\log\mathcal{E}$	0.023	**0.019**	0.236***	0.234***
	(0.038)	**(0.066)**	(0.088)	(0.072)
$(\pi\tilde{\alpha}-\tilde{\alpha})\,\Delta\log\mathcal{E}$	-0.263***	**-1.007*** **	-1.454***	-1.927***
	(0.082)	**(0.142)**	(0.188)	(0.160)
MSA fixed effects	✓	✓	✓	✓
# of firms	162,577	133,817	110,233	98,549
# of balanced workers	3,410,686	2,990,038	2,690,508	2,561,246

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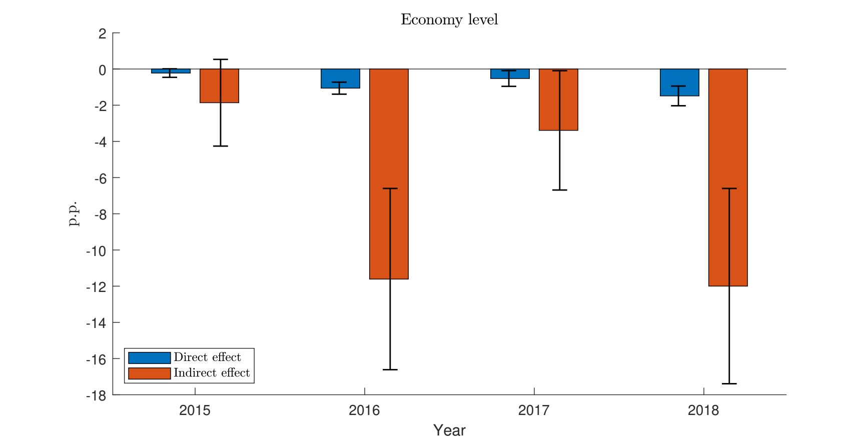
Implications for Aggregate Wage Adjustment

$$\mathbb{E}\left[\omega' \Delta \log W | \iota, \Delta Z\right] \sim \underbrace{\omega' {\color{red}\Delta Z} \hat{\beta}_1}_{\text{direct effect}} + \underbrace{\sum_{s=0}^{\infty} \hat{\lambda}^{s+1} \pi^{s+1} {\color{red}\Delta Z} \hat{\beta}_1 + \hat{\lambda}^{s} \pi^{s+1} {\color{orange}\Delta \tilde{Z}} \hat{\beta}_2}_{\text{indirect effect}},$$ where ω are weights Details



Concluding remarks

- Strong degree of strategic interactions in wage setting
  - **Endo strategic interactions**: Firms react to wage adjustment by their competitors (0.2-0.3)
  - **Exo strategic interactions**: Firms react to external shocks to their competitors
- Strategic interactions slow down aggregate wage adjustment by 10 p.p. over two years
- Additional results
  - Worker flows' in networks: homophily and workers' sorting.
  - Wages and employment adjustment under real and nominal rigidities

Appendix

Literature

- Strategic interactions in price and wage settings: Amiti et al. (2019), Staiger et al. (2010), Chan et al. (2024), Berger et al. (2022), Droste (2024)
  - Direct versus "industry" competitors
- Wage cyclicality: Bils (1985); Kudlyak (2014); Haefke et al. (2013); Haltiwanger et al. (2018); Gertler et al. (2020); Hazell and Taska (2020); Grigsby et al. (2021)
  - Role of strategic interactions in wage adjustment
- ER shocks on investment, employment, and productivity: Verhoogen (2008), Campa and Goldberg (1995), Nucci and Pozzolo (2001), Ekholm et al. (2012), Barbiero (2021), Blanco et al. (2022)
  - Propagation via network
- Peer effects and networks: Manski (1993), Bramoullé et al. (2009), de Paula (2017), Kuersteiner and Prucha (2020)
  - Peer effects at the economy level and implications for aggregate wage adjustment

Worker Flows Dynamics

- Worker Flows estimation *à la* Silva and Tenreyro (2006)

$$\mathbb{E}\left(\text{Flows}_{xt,j\to k} \middle| \chi_{jk,xt}\right) = \exp\left(\chi_{jk,xt}\right), \tag{4}$$

where $\chi_{jk,xt} = \sum_{s \in \left\{ \substack{\text{Firm pay} \\ \tilde{\gamma} \\ \tilde{\alpha}} \right\}} \tau_1^s \times (s_k - s_j) + \sum_{s \in \left\{ \substack{\text{Firm pay} \\ \tilde{\gamma} \\ \tilde{\alpha}} \right\}} \tau_2^s \times (s_k - s_j) \times \log \mathcal{E}_t + \text{Add Cntrls},$

- $\text{Flows}_{xt,j\to k}$ is total flow of x-type workers from j to k in year t
- Additional Controls include:
  - Employment
  - Firms' distance in: (i) age composition, (ii) gender composition, (iii) worker pay composition
  - Firm j and k fixed effects

Worker Flows Dynamics: Estimation Results

	(1)		(2)		(3)	
Firm pay$_k$ − Firm pay$_j$	0.803**	(0.332)	0.269	(0.223)	**1.545*****	**(0.304)**
$\tilde{\gamma}_k - \tilde{\gamma}_j$	-0.650*	(0.359)	-0.636*	(0.337)	**-0.202**	**(0.408)**
$\tilde{\alpha}_k - \tilde{\alpha}_j$	3.316***	(0.790)	2.205***	(0.919)	**15.905*****	**(1.025)**
$(\text{Firm pay}_k - \text{Firm pay}_j) \times \log \mathcal{E}_t$	-0.101**	(0.042)	-0.069**	(0.028)	**-0.090****	**(0.037)**
$(\tilde{\gamma}_k - \tilde{\gamma}_j) \times \log \mathcal{E}_t$	0.087*	(0.046)	0.092*	(0.048)	**0.098***	**(0.051)**
$(\tilde{\alpha}_k - \tilde{\alpha}_j) \times \log \mathcal{E}_t$	-0.352***	(0.100)	-0.409***	(0.116)	**-0.438*****	**(0.125)**
Firm j fixed effects			✓		✓	
Firm k fixed effects					✓	
MSA fixed effects	✓					
Year fixed effects	✓		✓		✓	
Employment	✓		✓		✓	
Distance	✓		✓		✓	
# of firm pairs	35,303,952		34,860,486		34,488,654	

Standard errors in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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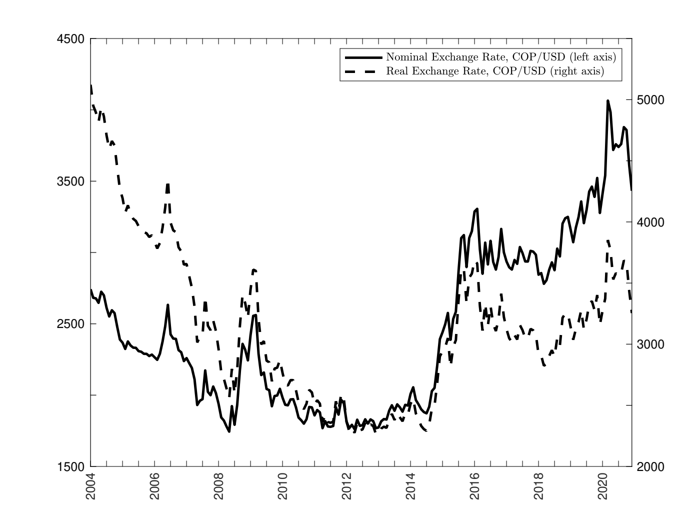
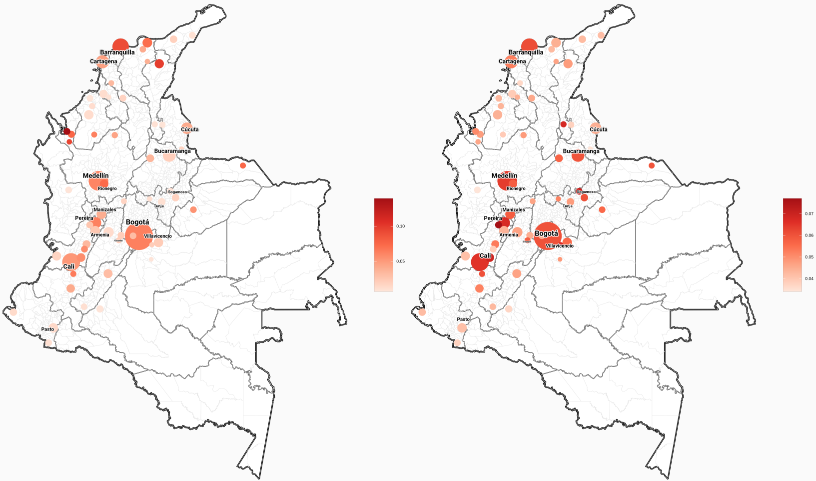
Appendix: Figures

Exchange Rate Dynamics



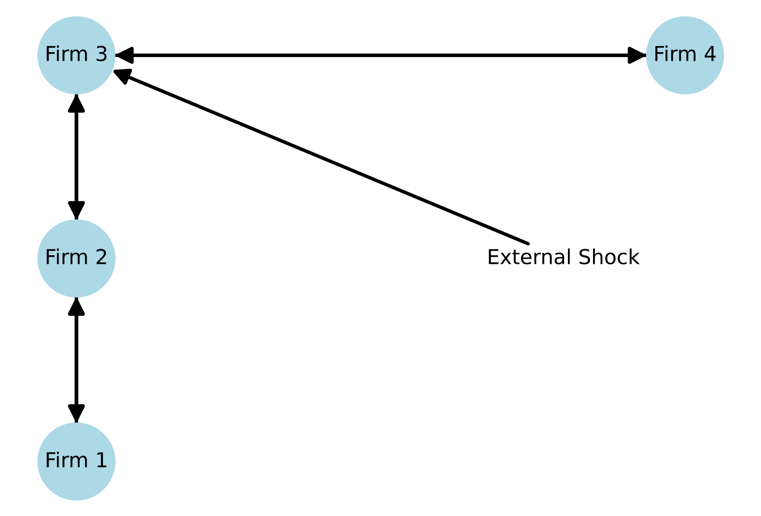
This paper
Data

Heatmap: Share of Trade with Abroad (left) and Share of Foreign Inputs (right)



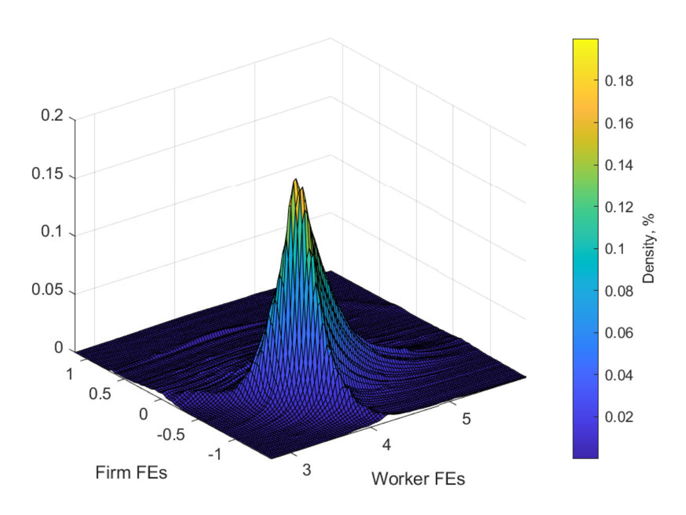
Data | Construction of Variables | This paper

Identification: Illustrative Example

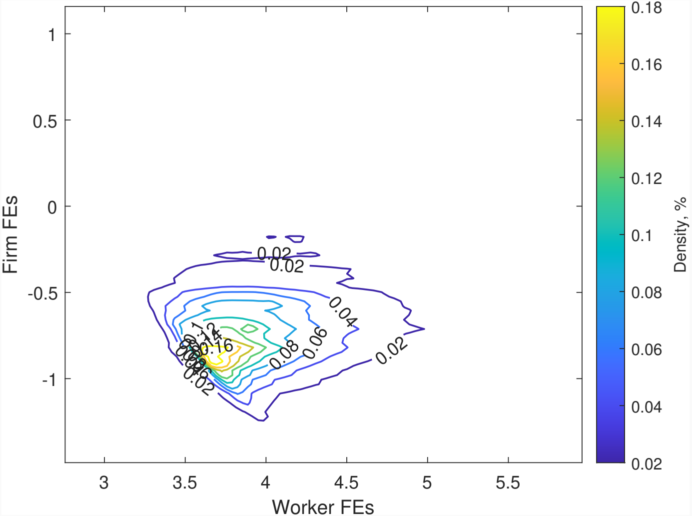


Back

Distribution of firm and worker effects



Distribution of firm and worker effects cont'd



Appendix: Model

Model: Workers

- Following Card et al. (2018); Lamadon et al. (2022), worker i of type x values firm j:

$$\mathcal{V}^{w}_{t,ij} = \phi_x \log W_{xjt} + \log V_{xj} + \log Z_{jt}\beta_z + \epsilon_{ij}, \tag{5}$$

  - W_{xjt} is wage of firm j at t
  - ϕ_x is degree of substitution across firms
  - V_{xj} is "amenities" of firm j
  - Z_{jt} is firm j's external shock(s) at t
    - Captures labor reallocation across firms
  - $\epsilon_{ij} \sim$ GEV distribution with "correlation" $\rho_x \in [0, 1)$ within a group
  - Back

Model: Final Good Producers

- Final good producers for Home and Rest of the World (RoW)

$$\mathcal{Y}_t = \left(\sum_{j=1}^{\mathcal{J}} \mu_j^{\frac{1}{\eta}} Y_{jt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{6}$$

$$\mathcal{Y}_t^* = \left(\sum_{j=1}^{\mathcal{J}} \mu_j^{*\,\frac{1}{\eta}} Y_{jt}^{*\,\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{7}$$

- μ_j and μ_j^* are weights firm j's goods used in production of goods for Home and ROW
- Therefore, demand for firm j's goods is:

$$Y_{jt} = \mu_j P_{jt}^{-\eta} \mathcal{Y}_t \tag{8}$$

$$Y_{jt}^* = \mu_j^* P_{jt}^{*\,-\eta} \mathcal{Y}_t^* \tag{9}$$

Model: Overview Model: Firms

Model: Firms

- Firm j produces goods according to production function

$$F_j\left(L_{jt}, M_{jt}\right) = \alpha_j^{-\alpha_j} L_{jt}^{1-\alpha_j} M_{jt}^{\alpha_j}, \tag{10}$$

$$L_{jt} = A_j \prod_{x=1}^{\mathcal{X}} L_{xjt}^{\beta_{xj}} \tag{11}$$

  - where L_{xjt} captures employed labor of type x by the firm
  - M_{jt} stands for quantity of foreign intermediate inputs used in production
  - α_j captures share of foreign intermediate inputs used in production

- Firm j sells goods at Home and to the Rest of Word (RoW) with demands:

$$Y_{jt} = \mu_j P_{jt}^{-\eta} \mathcal{Y}_t, \tag{12}$$

$$Y_{jt}^* = \mu_j^* P_{jt}^{*\,-\eta} \mathcal{Y}_t^*, \tag{13}$$

  - with P_{jt} and P_{jt}^* are prices for products of firm j invoiced in pesos and dollars, respectively

Strategic Interactions

- Total differential of equation (3):

$$
\begin{aligned}
\Delta \log W_{xjt} &= \tilde{\gamma}_j \Delta \log \mathcal{E}_t - \tilde{\alpha}_j \Delta \log \mathcal{E}_t && \text{external shocks} \\
&+ \sum_{k \neq j} \underbrace{\frac{\partial \log \mathcal{M}_{xjt}}{\partial \log W_{xkt}}}_{\sim L_{xkt}} \Delta \log W_{xkt} && \text{endo strategic interactions} \\
&+ \sum_{k \neq j} \underbrace{\frac{\partial \log \mathcal{M}_{xjt}}{\partial \log \mathcal{E}_t}}_{\sim L_{xkt}} \left(\tilde{\gamma}_k - \tilde{\gamma}_j\right) \Delta \log \mathcal{E}_t && \text{exo strategic interactions 1} \\
&+ \sum_{k \neq j} \underbrace{\frac{\partial \log \mathcal{M}_{xjt}}{\partial \left(\tilde{\alpha}_k - \tilde{\alpha}_j\right) \log \mathcal{E}_t}}_{\sim L_{xkt}} \left(\tilde{\alpha}_k - \tilde{\alpha}_j\right) \Delta \log \mathcal{E}_t, && \text{exo strategic interactions 2}
\end{aligned}
\tag{14}
$$

Model: Market Structure

- $\{H_{xjkt}\}_{k \neq j \text{ and } k,j \in \mathcal{J}_g}$ defines market structure

- Bertrand: $H_{xjkt} = W_{xkt} - W^{\#}_{xkt}$
  - Firms compete in wages assuming their competitors stick to their optimal wages

- Cournot: $H_{xjkt} = L_{xkt}(W_{xkt}, \mathbb{W}_{-xkt}) - L^{\#}_{xkt}$
  - Firms compete in quantities of labor assuming their competitors stick to their optimal quantities of labor

- Collusion: $H_{xjkt} = W_{xkt} - W_{xjt}$
  - Firms synchronize wage adjustment

- Back

Model: Households

- Households maximize:

$$\mathcal{V}_t\left(\mathcal{B}^*_{t-1}\right) = \max\left\{\log \mathcal{C}_t + \mathbb{E}_t \mathcal{V}_{t+1}\left(\mathcal{B}^*_t\right)\right\}, \tag{15}$$

- subject to the households' budget constraint:

$$\mathcal{P}_t \mathcal{C}_t + \mathcal{E}_t \mathcal{B}^*_t = \sum_{j=1}^{\mathcal{J}} \sum_{x=1}^{\mathcal{X}} W_{xjt} L_{xjt} + \mathcal{E}_t \mathcal{O}_t + \left(1 + i^*_{t-1}\right) \mathcal{E}_t \mathcal{B}^*_{t-1} + \text{Profit}_t, \tag{16}$$

- $\mathcal{C}_t = \left(\frac{\mathcal{C}_{Ht}}{\omega}\right)^{\omega} \left(\frac{\mathcal{C}_{Ft}}{1-\omega}\right)^{1-\omega}$ and $\mathcal{P}_t \mathcal{C}_t \equiv \mathcal{C}_{Ht} + \mathcal{E}_t \mathcal{C}_{Ft}$ represents households' expenditure on domestic and foreign goods
- $\mathcal{B}^*_{t-1}$ defines net foreign assets carried over from period $t-1$, and i^*_t stands for interest rate on net foreign assets.
- $\mathcal{O}_t$ is endowment of oil in dollars, and Profit_t is a lump-sum profit of intermediate good producers.

Model: Overview

Model: Market Clearing Condition

- Home goods:

$$\mathcal{C}_{Ht} = \mathcal{Y}_t \tag{17}$$

- Balance of Payments:

$$\mathcal{B}_t^* = \mathcal{Y}_t^* + \mathcal{O}_t - \mathcal{M}_t - \mathcal{C}_{Ft} + \left(1 + i_{t-1}^*\right) \mathcal{B}_{t-1}^* \tag{18}$$

- Oil endowment:

$$\log \mathcal{O}_t - \log \bar{\mathcal{O}} = \rho_o \left(\log \mathcal{O}_{t-1} - \log \bar{\mathcal{O}}\right) + \epsilon_{ot}, \tag{19}$$

- Interest rate on net foreign assets:

$$i_t^* = \bar{i}^* + \psi \, \frac{\mathcal{B}_t^* - \bar{\mathcal{B}}^*}{\bar{\mathcal{B}}^*} + \epsilon_{i^* t}, \tag{20}$$

- Model: Overview

Model: Equilibrium definition

A competitive equilibrium is a set of final good producers' prices $\mathbb{P}_t = \{P_{jt}\}_{j=1}^{\mathcal{J}}$ and $\mathbb{P}^* = \{P_{jt}^*\}_{j=1}^{\mathcal{J}}$, firms' wages $\mathbb{W}_{xt} = \{W_{xjt}\}_{j=1}^{\mathcal{J}}$ for each type x, decision rules $\{L_{xjt}(\mathbb{W}_{xt})\}_{x,j}$, $\mathbb{Y}_t = \{Y_{jt}\}_{j=1}^{\mathcal{J}}$, $\mathbb{Y}_t^* = \{Y_{jt}^*\}_{j=1}^{\mathcal{J}}$, household's choices $\{\mathcal{C}_t, \mathcal{C}_{Ht}, \mathcal{C}_{Ft}, \mathcal{B}_t^*\}$ such that, given firms' productivities $\mathbb{A} = \{A_j\}_{j=1}^{\mathcal{J}}$, and international shocks $\mathcal{O}_t$ and i_t^*,

1. The firms' wages $\mathbb{W}_{xt} = \{W_{xjt}\}_{j=1}^{\mathcal{J}}$ solve the intermediate good producer's problem (2),
2. Workers choose firms according to (**??**) that results in the labor supply (1).
3. The demand for each intermediate good producer's output from the final good producers Y_{jt} and Y_{jt}^* is equal to the supply of that firm's output $F_j(L_{jt}, M_{jt})$,
4. Household smooths consumption; that is: $1 = (1 + i_t^*)\beta\mathbb{E}_t\left\{\frac{\mathcal{C}_t}{\mathcal{C}_{t+1}}\frac{\mathcal{P}_t}{\mathcal{P}_{t+1}}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right\}$,
4. The home goods market clears; that is: $\mathcal{Y}_t = \mathcal{C}_{Ht}$,
5. The balance of payment holds; that is: $\mathcal{B}_t^* = \mathcal{Y}_t^* + \mathcal{O}_t - \mathcal{M}_t - \mathcal{C}_{Ft} + (1 + i_{t-1}^*)\mathcal{B}_{t-1}^*$, where

   $\mathcal{M}_t = \sum_{j=1}^{\mathcal{J}} M_{jt}$ and $\mathcal{C}_{Ft} = (1-\omega)\mathcal{E}_t^{-\omega}\mathcal{C}_t$,

6. The labor markers clear $\sum_{j=1}^{\mathcal{J}} L_{xjt} = 1$ for each x.

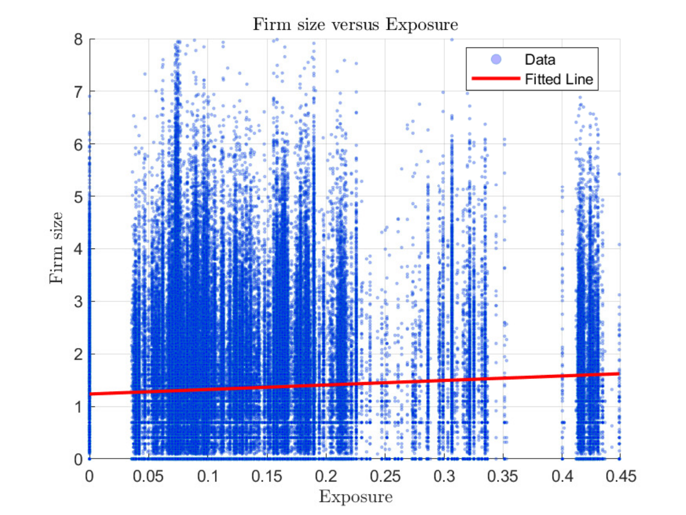
Model: Overview

Appendix: Calibration

Parameter	Description	Value	Target / Source
β	Discount factor	0.96	Annual interest rate $r = 4.0\%$
ω	Share of Home goods	0.90	Balance of Payment condition
α_j	Share of foreign intermediate inputs		IO matrix
μ_j	Domestic demand		IO matrix
μ_j^*	Foreign demand		IO matrix
$\bar{\mathcal{O}}$	Commodity endowment	0.096	10 % of GDP
$\bar{\mathcal{B}}^*$	Foreign debt	0.298	31 % of GDP
η	Elasticity of substitution	4.0	Broda & Weinstein (2006)
ψ	Foreign interest rate elasticity	0.01	Value enough to close model
ρ_o	Persistence of commodity shocks	0.7	Gopinath et al. (2020)
$\bar{i}^*$	Foreign interest rate (s.s. value)	4%	Annual interest rate $r = 4.0\%$
$\mathcal{J}$	Number of firms	61	2-digit sectors
ϕ_x	Labor supply elasticity	1.0	Imposed
ρ_x	Correlation within a group	0.5	Imposed
V_{xj}	Unobservable firm amenities		Network estimation

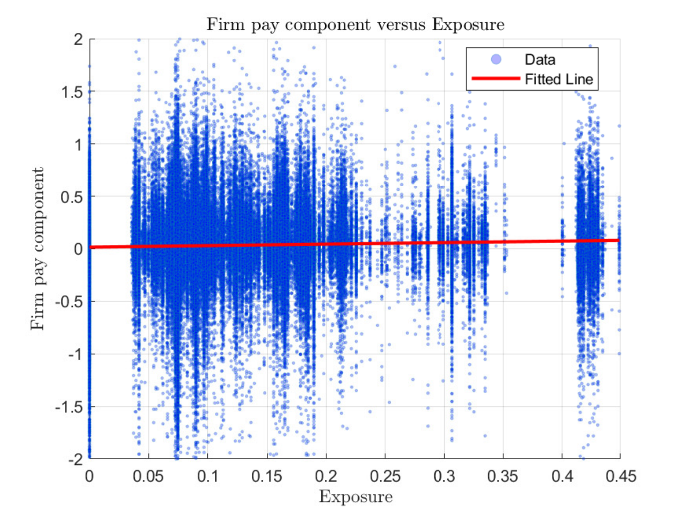
Notes: In our calibration, time period is a year. Model: Overview

Firm Size Job Ladders and Exposure: Entire economy



Back

Firm Pay Job Ladders and Exposure: Entire economy



Back

Appendix: Additional tables

Summary statistics and AKM results

	All	Above minimum wage	Above minimum wage - AKM
Sample size			
# of worker-years	72,447,390	61,955,631	61,447,455
# of workers	14,077,932	13,741,240	13,559,020
# of firms	829,037	727,749	599,458
Summary statistics			
Mean of log wages	3.48	3.57	3.57
Var of log wages	0.37	0.38	0.38
Share of variance of wages explained by each component			
MSA-firms			0.13
Workers			0.59
Xb			0.12
Overall fit of AKM decomposition			
Adj. R^2			0.8647

AKM Data

Metropolitan Areas (in 2018)

Metropolitan area	Frequency	Percent	Cumulative
Bogota	4,447,169	35.17	35.17
Medellin	2,027,496	16.12	51.49
Cali	1,217,315	9.68	61.17
Barranquilla	710,773	5.65	66.82
Cartagena	349,734	2.78	69.61
Bucaramanga	496,883	3.95	73.56
Cucuta	217,365	1.73	75.29
Pepeira	283,280	2.25	77.54
Sogamoso	35,065	0.28	77.82
Rionegro	96,729	0.77	78.59
Tunja	113,025	0.90	79.49
Armenia	138,816	1.10	80.59
Others		19.14	100.00

Data

Industry Coverage (in 2018)

	Frequency	Percent	Cumulative
Agriculture, Forestry, and Fishing	700,258	4.96	4.96
Mining	124,664	0.88	5.84
Construction	1,346,825	9.53	15.37
Manufacturing	1,228,695	8.70	24.07
Transportation, Communications, Electric, Gas, and Sanitary Services	809,363	5.73	29.80
Wholesale and Retail Trade	1,585,878	11.23	41.03
Finance and Real Estate	4,844,528	34.30	75.33
Services	1,889,030	13.36	88.69
Education	764,089	5.41	94.10
Public Administration	834,022	5.90	100.00

Data

Appendix: Estimation details

Firm and worker pay components

- Firm and worker pay components as in Abowd, Kramarz, and Margolis (1999) (AKM)
- Baseline specification:
$$\log W_{it} = \lambda_i + \psi_{J(i,t)} + X_{it} b + \varepsilon_{it}, \tag{21}$$
    - $\log W_{it}$ is log of daily wage in 1,000s of peso of worker i in period t
    - $J(i, t)$ is a firm worker i is in at period t
    - X_{it} includes unrestricted MSA-time fixed effects along with quadratic and cubic functions of age of worker i
    - AKM results Data

Implications for Wage Adjustment

- Wage adjustment in the matrix form:

$$\Delta \log W = \iota + \lambda \pi \Delta \log W + \Delta Z \beta_1 + \pi \Delta \tilde{Z} \beta_2 + u \tag{22}$$

  - ΔZ is a $\mathcal{J} \times 2$ matrix with j-th row $[\tilde{\gamma}_j \Delta \log \mathcal{E}_t, \tilde{\alpha}_j \Delta \log \mathcal{E}_t]$
  - $\Delta \tilde{Z}$ is a $\mathcal{J} \times 2$ matrix with j-th row $\left[\sum_{k \neq j} \pi_{x,jk} \left(\tilde{\gamma}_k - \tilde{\gamma}_j\right) \Delta \log \mathcal{E}_t, \sum_{k \neq j} \pi_{x,jk} \left(\tilde{\alpha}_k - \tilde{\alpha}_j\right) \Delta \log \mathcal{E}_t\right]$

- Spatial spillovers:

$$\mathbb{E}\left[\Delta \log W | \iota, \Delta Z\right] \propto \underbrace{\Delta Z \beta_1}_{\text{Direct effects}} + \sum_{s=0}^{\infty} \underbrace{\lambda^{s+1} \pi^{s+1} \Delta Z \beta_1 + \lambda^{s} \pi^{s+1} \Delta \tilde{Z} \beta_2}_{\text{Indirect effects}} \tag{23}$$

Implications for Aggregate Wage Adjustment

Strategic Interactions in Wage Setting: Identification

- Specification

$$\Delta \log W = \iota + \lambda\pi\Delta \log W + \Delta Z\beta_1 + \pi\Delta\tilde{Z}\beta_2 + u$$

- Linear instruments (if $I - \lambda\pi$ is invertible):

$$\mathbb{E}\left[\pi\Delta \log W | \Delta Z\right] = \pi\left(I - \lambda\pi\right)^{-1}\Delta Z\beta^* = \sum_{s=0}^{\infty}\lambda^s\pi^{s+1}\Delta Z\beta^*$$

- Quadratic instruments:

$$\mathbb{VC}\left[\Delta \log W | \iota, \Delta Z\right] = \left(I - \lambda\pi\right)^{-1}\left(I - \lambda\pi\right)^{-1}\sigma_u^2 = \sum_{s=0}^{\infty}\sum_{\tau=0}^{\infty}\lambda^{s+\tau}\pi^s{\pi'}^{\tau}\sigma_u^2$$

Back

Strategic Interactions in Wage Setting: Identification

- We use linear and quadratic moments:

$$\mathbb{E}\left[(\hat{\pi}\Delta z)' u\right] = 0$$

$$\mathbb{E}\left[\left(\hat{\pi}^2\Delta\tilde{z}\right)' u\right] = 0$$

$$\mathbb{E}\left[u'\hat{\mathcal{A}}^1 u\right] = 0$$

$$\mathbb{E}\left[u'\hat{\mathcal{A}}^2 u\right] = 0$$

- with $\hat{\mathcal{A}}^1 = (\hat{\pi} + \hat{\pi}')/2$ and $\hat{\mathcal{A}}^2 = \hat{\pi}\hat{\pi}' - \mathrm{diag}(\hat{\pi}\hat{\pi}')$
- The quadratic moments alone lead to ($q = 1, 2$)

$$\mathbb{E}\left[(\pi\Delta \log W)' \hat{\mathcal{A}}^q u\right] = 0$$

Back